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BOOK OF ABSTRACTS

Nonlinear PDEs at Valparaíso

a conference in honor of Patricio Felmers 60th anniversary.

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Ground states of elliptic equations with competition between power and gradient terms

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Here we consider the nonnegative solutions of equations in a punctured ball $B(0,R)\setminus\{0\}\subset\mathbb{R}^N$ or in $\mathbb{R}^N$, of type

$$-\Delta u = u^p + M|\nabla u|^q \tag{0.1}$$

where $p, q > 1$ and $M \in \mathbb{R}$. We give new a priori estimates on the solutions and their gradient, and Liouville type results. We use Bernstein technique and Osserman's or Gidas-Spruck's type methods. The most interesting case is $q = 2p/(p+1)$, where the equation is invariant by scaling. In the radial case, we give a precise description of all the regular and singular solutions, improving the known results. The situation appears to be quite complicated in the case $M < 0$ of strong competition between the nonlinear terms.

A class of very degenerate operators: existence through convexity

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In a recent paper with Galise and Ishii we have proved existence of solutions for the Truncated Laplacians, a class of very degenerate fully nonlinear operators, when the domain is hula hoop i.e. uniformly convex and the first order term is small. We shall see which of these conditions can be removed and which can't.

On isolated singularities for elliptic equation with Hardy operator.

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In this talk, we would like to talk about the isolated singularities for Hardy operators:

• the fundamental solutions of Hardy operator

$$\mathcal{L}_\mu := -\Delta - \frac{\mu}{|x|^2};$$

• qualitative properties of the solution of nonhomogeneous of

$$\mathcal{L}_\mu u = f \quad \text{in} \quad \Omega \setminus \{0\}, \qquad u = 0 \ \text{ on } \ \partial\Omega;$$

• the classification, existence and nonexistence of the isolated singularities of semilinear Hardy equation

$$\mathcal{L}_\mu u = u^p \quad \text{in} \quad \Omega \setminus \{0\}, \qquad u = 0 \ \text{ on } \ \partial\Omega.$$

• Application to semilinear linear elliptic equation $-\Delta u = Vf(u)$ in exterior domain.

Large time behavior of solutions of the porous medium equation in exterior domains.

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Let $\mathcal{H} \subset \mathbb{R}^N$ be a non-empty bounded open set. We consider the porous medium equation in the complement of $\mathcal{H}$, with zero Dirichlet data on its boundary and non-negative compactly supported integrable initial data.

Kamin and Vázquez, in 1991, studied the large time behavior of solutions of such problem in space dimension 1. Gilding and Goncerzewicz, in 2007, studied this same problem dimension 2. Using their results in the outer field we study the large time behavior of the solution in the near field scale, in particular in bounded sets of $\mathbb{R}^N \setminus \mathcal{H}$.

This a joint work with Fernando Quirós (Universidad Autonoma de Madrid, Spain) and Noem Wolanski (Universidad de Buenos Aires, Argentina).

TBA.

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Vortex dynamics in Euler flows and the Liouville equation .

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We consider the two-dimensional Euler flow for an incompressible fluid confined to a smooth domain. We construct smooth solutions with concentrated vorticities around $/itkk$ points which evolve according to the Hamiltonian system for the Kirkhoff-Routh energy. The profile around each point resembles a scaled finite mass solution of Liouville's equation. This is joint work with Juan Davila, Monica Musso and Juncheng Wei.

TBA.

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Symmetry and symmetry breaking in PDEs.

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Symmetry and symmetry breaking phenomena in elliptic PDEs are essential for the classification of the solutions and for applications for instance in physics. While symmetry breaking can often be proved by linear instability of symmetric solutions, symmetry results are global and require more elaborate tools. Some insight can be gained by using appropriate flows and monotonicity properties of related functionals. The lecture is intended to give a partial review on this topic and will in particular cover some of the results on models with magnetic fields obtained in collaboration with Maria J. Esteban, and several other co-authors.

Interior regularity results for zero-th order operators approaching the fractional Laplacian.

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In this lecture we going to talk about interior regularity results for the solution $u_\epsilon \in C(\bar{\Omega})$ of the Dirichlet problem

$$\begin{cases} -\mathcal{I}_\epsilon(u) = & f_\epsilon \qquad \text{in } \Omega \\ \qquad u = & 0 \qquad \text{in } \Omega^c. \end{cases} \quad . \tag{0.2}$$

where $-\mathcal{I}_\epsilon$ is an approximation of the well-known fractional Laplacian of order σ,as ϵ tends to zero. The purpose of this talk is to understand how the interior regularity of u_ϵ evolves as ϵ approaches zero. We going to present recent results which provide that u_ϵ has a modulus of continuity which depends on the modulus of f_ϵ, which becomes the expected Hölder profile for fractional problems, as $\epsilon \to 0$. This analysis includes the case when f_ϵ deteriorates its modulus of continuity as $\epsilon \to 0$.

Joint work with P. Felmer (CMM-DIM) and E. Topp (USACH).

Magnetic interpolation inequalities in dimensions 2 and 3.

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In this talk I will present various results concerning interpolation inequalities, best constants and information about the extremal functions concerning Schrdinger magnetic operators in dimensions 2 and 3. The particular, and physical interesting cases of constant and of Aharonov-Bohm magnetic fields will be discussed in detail.

These works have been made in collaboration with J. Dolbeault, A. Laptev and M. Loss.

On the existence and uniqueness of bound state solutions of a semilinear equation with weights.

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We consider radial solutions of a general elliptic equation involving a weighted Laplace operator. We establish the existence and uniqueness of sign changing solutions to

$$\operatorname{div}(A\nabla v) + Bf(v) = 0, \ \lim_{|x|\to\infty} v(x) = 0, \ x \in \mathbb{R}^n, \tag{0.3}$$

$n > 2$, where A and B are two positive, radial, smooth functions defined on $\mathbb{R}^n \setminus \{0\}$. We assume that the nonlinearity $f \in C(-c, c), \ 0 < c \le +\infty$ is an odd function satisfying some convexity and growth conditions, and has a zero at $b > 0$, is non positive and not identically 0 in $(0, b)$, positive in (b, c), and is differentiable in $(0, c)$.

Joint work with C. Cortázar and P. Herreros (PUC, Chile).

The vanishing discount problem for Hamilton-Jacobi equations in Euclidean n space .

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I will present recent joint work with A. Siconolfi which concerns the vanishing discount problem for Hamilton-Jacobi equations in Euclidean n space. Under appropriate assumptions, which, in particular, imply the compactness of the (projected) Aubry set of the associated ergodic problem, the convergence of the whole family of solutions of the discount problems, as the discount factor tends to zero, is established.

Liouville theorems for radial solutions of semilinear elliptic equations.

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In this work we obtain some new Liouville theorems for positive, radially symmetric solutions of the equation

$$-\Delta u = f(u) \qquad \text{in } \mathbb{R}^N$$

where f is a continuous function in $[0, +\infty)$ which is positive in $(0, \infty)$. Our methods adapt to cover more general problems, where the nonlinearity is multiplied by some radially symmetric weights and/or the Laplacian is replaced by the p-Laplacian, $1 < p < N$. Some results for related elliptic systems are also obtained.

Multiple steady states for a competition system supporting an ideal-free distribution.

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In this talk we will discuss existence of steady state solutions for the competitive system

$$\begin{cases} \dfrac{\partial u}{\partial t} = \nabla \cdot \left[\alpha(x)\nabla \dfrac{u}{m}\right] + u(m(x) - u - bv) \ \text{ in } \Omega, \ t > 0, \\ \\ \dfrac{\partial u}{\partial t} = \nabla \cdot [\beta(x)\nabla v] + v(m(x) - cu - v) \ \text{ in } \Omega, \ t > 0, \\ \\ \nabla \dfrac{u}{m} \cdot \hat{n} = \nabla v \cdot \hat{n} = 0 \ \text{ on } \partial\Omega, \ t > 0, \end{cases}$$

which supports an *ideal free distribution* for the first species, i.e. admits a positive steady state which matches the per-capita growth rate. Previous results have stated that when $b = c = 1$ the ideal free distribution is an evolutionary stable and neighborhood invader strategy, that is the species with density v always goes extinct. We will analyze how the interaction coefficients b and c influence the structure of the steady state solutions of the system. In particular, how and to what extent the advantage derived from ideal free dispersal continues when there is a trade off relative to competitive impact, for example when $b > 1$ but $c < 1$. To understand this case, we will study the interplay between the inter-specific competition coefficients b, c and the diffusion coefficients $\alpha(x)$ and $\beta(x)$ on the critical values for stability of semi-trivial steady states. We will also show that under certain regimes the system sustains multiple positive steady states.

Games for eigenvalues of the Hessian and concave/convex envelopes.

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We deal with the PDE $\lambda_j(D^2u) = 0$, in Ω, with $u = g$, on $\partial\Omega$. Here $\lambda_1(D^2u) \leq ... \leq \lambda_N(D^2u)$ are the ordered eigenvalues of the Hessian D^2u. The equation $\lambda_1(D^2u) = 0$ is just the PDE verified by the convex envelope inside Ω of the boundary datum g. Our main result is to show a necessary and sufficient condition on the domain so that the problem has a continuous solution for every continuous datum g. We also introduce a related two-player zero-sum game whose values approximate solutions to this PDE problem.

Uniform boundedness of positive solutions of the Lane-Emden equation in dimension two .

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We prove that positive solutions of the Lane-Emden equation in a two-dimensional smooth bounded domain are uniformly bounded for all large exponents. A consequence is an integral bound which implies sharp asymptotic for such solutions. Joint work with Nikola Kamburov.

Standing waves for Chern-Simons-Schrödinger systems.

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In this talk we prove existence and Concentration of standing waves for the Chern-Simons-Schrödinger System with general nonlinearity by using variational methods.

Singular perturbation problem for generalized Choquard equations.

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We study the singular perturbation problem for for a class of generalized Choquard equations:

$$-\varepsilon^2 \Delta u + V(x)u = \frac{1}{\varepsilon^\alpha}(I_\alpha * F(u))f(u) \quad \text{in } \mathbb{R}^N,$$

We study the existence and multiplicity of concentrating solutions using variational methods. Especially, we give an existence result for locally sublinear case.

This is a joint work with S. Cingolani.

Fractional Dirichlet problems with supercritical gradient terms.

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In this talk we present existence and uniqueness results for Dirichlet problems associated to fractional elliptic equations with the form

$$(-\Delta)^s u = g(|Du|) + f \quad \text{in } \Omega, \tag{0.4}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with smooth boundary, $f \in C(\bar{\Omega})$, $s \in (0,1)$, and g is a coercive nonlinearity with a supercritical growth with respect to the fractional diffusion, that is $g(t)/t^{2s} \to +\infty$ as $t \to +\infty$.

We construct suitable barriers in order to get the existence and uniqueness of solutions for some Dirichlet problems associated to (0.4), attaining continuously the boundary data.

Joint wotk with A. Quaas and G. Dávila (UTFSM).

Non-linear Schrödinger equation with non-local regional diffusion.

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In this work we are interested in the nonlinear Schrödinger equation with non-local regional diffusion

$$\epsilon^{2\alpha}(-\Delta)^{\alpha}_{\rho}u + u = f(u) \;\text{ in }\; \mathbb{R}^n \qquad (0.5)$$
$$u \in H^{\alpha}(\mathbb{R}^n)$$

where f is a continuous function with suitable conditions and $(-\Delta)^{\alpha}_{\rho}$ is a variational version of the regional laplacian, whose range of scope is a ball with radius $\rho(x) > 0$. We give general conditions on ρ which assure the existence and multiplicity of solution for the problem (0.5). Furthermore, we study the behavior of semi-classical solutions as $\epsilon \to 0$.

Existence of positive solutions of Schrödinger equations with vanishing potentials.

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In this talk we show the existence of at least one positive solution of the equation

$$\begin{cases} -\Delta u + V(x)u = f(x,u), \quad x \in \mathbb{R}^N \\ u > 0 \\ u \in H^1(\mathbb{R}^N), \end{cases} \tag{0.6}$$

for $N \geq 3$ and assuming that $f : \mathbb{R}^N \times \mathbb{R} \to \mathbb{R}$ is a continuous nonnegative function and $V : \mathbb{R}^N \to \mathbb{R}$ is a nonnegative potential. Here, we assume that f is superlinear at the origin and at infinity and has subcritical growth, and we give examples where f does not satisfy the classical condition of Ambrosetti-Rabinowitz nor monotonicity conditions. Also, we consider situations where the potential $V : \mathbb{R}^N \to \mathbb{R}$ can vanish at infinity.

This is a joint work with Eduard Toon.

Elliptic equations with measure valued nonlinear absorption and measure data.

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We study the semilinear elliptic equation $-\Delta u + g(u)\sigma = \mu$ with Dirichlet boundary condition in a smooth bounded domain where σ is a nonnegative Radon measure, μ a Radon measure and g is an absorbing nonlinearity. We show that the problem is well posed if we assume that σ belongs to some Morrey class. Under this condition we give a general existence result for any bounded measure provided g satisfies a subcritical integral assumption. We study also the supercritical case when $g(r) = |r|^{q-1}r$, with $q > 1$ and μ satisfies an absolute continuity condition expressed in terms of some capacities involving σ. We give applications to the study of harmonic functions in a half-space satisfying a nonlinear Neumann boundary condition with measure data on the boundary

Hamilton-Jacobi systems involving Caputo time derivatives.

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In this talk we state existence and uniqueness of bounded viscosity solutions of weakly coupled systems of parabolic HamiltonJacobi equations with nonlocal ingredients, where the time evolution of each equation is driven by Caputo derivatives of different orders. As an application, we present steady-state large time behavior for the system in the case the stationary equation has uniqueness.

Qualitative properties of positive solutions for mixed integro-differential equations.

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In this talk, we introduce the qualitative properties of solutions to mixed integro-differential equation

$$\begin{cases} (-\Delta)^{\alpha}_{x} u + (-\Delta)_{y} u + u = f(u) & \text{in } \mathbb{R}^N \times \mathbb{R}^M, \\ u > 0 \ \text{ in } \mathbb{R}^N \times \mathbb{R}^M, & \lim_{|(x,y)| \to +\infty} u(x,y) = 0, \end{cases} \tag{0.7}$$

with $N \geq 1$, $M \geq 1$ and $\alpha \in (0,1)$. We study decay and symmetry properties of the solutions to this equation. Difficulties arise due to the mixed character of the integro-differential operators. Here, a crucial role is played by a version of the Hopf's Lemma we prove in our setting. In studying the decay, we construct appropriate super and sub solutions and we use the moving planes method to prove the symmetry properties.

Joint work with Prof. Patricio Felmer.